GPS - GPS - If I may - What's the distance I went today ?

Seeing REI's ad for discounts on GPS devices brought this thought back to my mind : We might be mistaken about the accuracy of the distance our GPS tells us we have gone.

The perhaps sad fact to realize is that your GPS must on average record a distance-traveled which is farther than you have actually gone. The proof of this should be obvious with the simple geometry which will be considered here, and provable with far more complex calculus which I will not attempt. The only real purpose of this is to let you know that the race director probably did not cheat you about how far you went from Start to Finish.

The error incurred is highly variable and is dependant upon the positional accuracy of the GPS device, upon the divergence of your actual path from a straight line, and upon the distance traveled between sampling points. The distance traveled between sampling points depends upon your traveling speed and the sampling rate of the GPS.

All of the (probably boring) hypothesis thoughts* given far below mean that while you actually travel from point-to-point, the GPS believes that you have traveled from and to somewhere inside two error-spheroids. While geometry and calculus would give a truly math-based answer, I will stick with a simple approximation using flat geometry. I will further simplify it by ignoring both the forward-backward error (since this will on average about cancel itself out), and also the vertical error (because this math would make my head hurt even more). I will use the more common 5 meter specification device for this discussion - if you spent many hundreds of dollars on a 1 meter unit your distance will be significantly more accurate, but still wrong.

Sampling rate is possibly a preference setting you can change on your device. If so, you might seriously think about how to set it. At first consideration, you might think that a higher sampling rate of say once per second would be better, but it can actually increase the distance error significantly. If you are walking at a basic 20 minutes per mile pace, you will only travel about 4.4 feet in the one second between sample points, but the positional error is still plus or minus about 16 feet! Using geometric approximations and averages (given below), your GPS will on average tell you that the distance traveled was about 9.1 feet, an error of about 200%. BUT, if your sampling rate is only once per 10 seconds, then the real distance is about 44 feet and the reported approximation is about 44.7 feet, an error of only about 1.6%.

The only reason to have a quick sampling rate I can think of is if you are traveling very fast (car, etc) or are on a very twisty route - one that often cuts back on itself in much less distance than you travel between sampling points. But even actually running (8-12 feet per second) on a sometimes very-twisty trail won't introduce much negative error.

All in all, the GPS units are marvelous at giving us quick and easy approximations of how far we went, and possibly life-saving information about where we are. Just remember to be fair to the race director - they might have had to walk the whole course with a measuring wheel.

*** Hypothesis thoughts :**

The positional accuracy of GPS devices ranges hugely, and is basically cost-based. Some expensive citizen-units are available which are much more accurate (perhaps within a 6 foot (2m) diameter sphere), but most can only place you somewhere inside a 32 foot (10m) diameter sphere. I have only considered the complicated-enough simplified horizontal circle in this discussion because the reality of the spheroid error, including the vertical and forward-backward components, makes the GPS distance-covered measurement error even worse - and the math is way too involved.

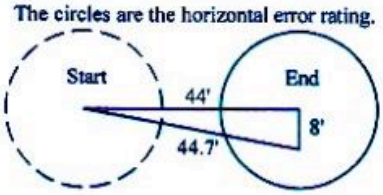
The accuracy rating of the device, if I understand it correctly, is a value which the device will meet 50% of the time; this means that its plus-minus rating is not the limit, it is only the 50% limit. Also noteworthy is that these ratings are only valid when the GPS has a clear view of many satellites. If you are down in the canyon, your GPS might place you way out where Wile E Coyote often found himself - Mbeep-Mbeep.

WAAS enabled devices certainly improve the overall navigational position accuracy, but the plus-minus error always exists.

The geometry used for the approximations above was to assume a right-triangle with the adjacent leg as the real distance traveled (speed and sampling rate), the opposite leg as half of the GPS accuracy rating (8'), and the hypotenuse as the distance the GPS reports.

Simplify by starting at the actual center of the error spheroid.
The straight horizontal lines are the actual paths travelled.
The straight vertical lines are one-half of the error rating.
The angled lines are the distance the GPS thinks you travelled.
<< Use Pythagorean's therom to get 9.13' from 4.4' per second, and one second of travel time.
Use Pythagorean's therom to get 44.7' from 4.4' per second, and ten second of travel time. >>

The circles are the horizontal error rating.



So, buy and use your GPS, and enjoy its valuable information. Just remember to take it all with a grain of salt - kinda like Wile E would like his Road Runner served for dinner. =})